ZiF-MS-02/96
FSU-SCRI-96C-122
UK/96-14

REDUCING DISCRETIZATION ERRORS IN LATTICE QCD SPECTROSCOPY[a]

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The improved Wilson quark action – the clover action – is constructed to have smaller discretization errors than the normal Wilson quark action. We test this in a quenched spectroscopy computation on 6 lattice ensembles with spacings from 0.15 to 0.43 fm. To ensure that the dominant scaling violations come from the fermions we use an $\mathcal{O}(a^2)$ improved 6-link $SU(3)$ pure gauge action. We find evidence that fermionic scaling violations are consistent with $\mathcal{O}(a^2)$ for clover fermions and $\mathcal{O}(a)$ with a nonnegligible $\mathcal{O}(a^2)$ term for standard Wilson fermions. This latter mixed ansatz makes a reliable continuum extrapolation problematic for Wilson fermions. For clover fermions, on the other hand, we obtain accurate predictions for hadron masses in quenched continuum QCD. We find that the slopes of the scaling violations are roughly 200 MeV for both Wilson and clover fermions.

1 Introduction

The goal of lattice QCD spectroscopy computations is, of course, to make mass predictions for *continuum* QCD. This requires in the end an extrapolation from results at finite lattice spacing a to the continuum limit, $a = 0$. We call the deviations of the finite lattice spacing results from their continuum limit value discretization errors. For Wilson fermions, the leading discretization errors are known to be of order $\mathcal{O}(a)$. In other words, relative discretization errors are expected to be about $a\Lambda_{\rm QCD}$, with $\Lambda_{\rm QCD} \approx 300 - 500$ MeV a typical QCD scale. In a typical simulation today, say at $\beta = 6$, where $a^{-1} \simeq 2$ GeV, we therefore expect discretization errors for computations with Wilson fermions

[a] Talk given by UMH at the Conference "Multi-Scale Phenomena and Their Simulation", September 30 – October 5, 1996, Bielefeld, Germany. To appear in the proceedings.

of 15 − 25%. Even at the smallest lattice spacing reached to date, the errors are expected to be $\gtrsim 10\%$.

Decreasing the lattice spacing to make the discretization errors smaller is a costly enterprise. Just the cost to create pure gauge configurations of constant physical volume grows like a^{-5}, and the cost to perform quenched spectroscopy computations at constant pion to rho mass ratio on these configurations grows probably even faster. For full QCD simulations the situation is even worse.[1,2] Therefore it seems worthwhile to 'improve' the action so as to reduce the *order* of the discretization errors from $\mathcal{O}(a)$ to $\mathcal{O}(a^2)$, as will be described in this talk, or even to higher order.[3]

2 The Symanzik improvement program

Symanzik has first proposed that lattice effects, leading to discretization errors, can be eliminated, order by order in a, by including irrelevant, higher dimension operators into the lattice action.[4] For example, the simplest pure gauge lattice action, the so-called Wilson action

$$S = \frac{\beta}{N} \sum_p \mathrm{ReTr}(1 - U_p)$$

becomes, as $a \to 0$

$$S \to \frac{1}{2} a^4 \sum_{x,\mu,\nu} \left\{ \mathrm{Tr}(F_{\mu\nu}^2) - \frac{1}{12} a^2 \mathrm{Tr}(D_\mu F_{\mu\nu} D_\mu F_{\mu\nu}) + \mathcal{O}(a^4) \right\} .$$

The $\mathcal{O}(a^2)$ contribution can be canceled by adding a 2×1-loop term

$$S = \frac{\beta}{N} \left\{ \frac{5}{3} \sum_p \mathrm{ReTr}(1 - U_p) - \frac{1}{12} \sum_{2\times 1} \mathrm{ReTr}(1 - U_{2\times 1}) \right\} .$$

Loop corrections (quantum effects) modify the coefficients, $5/3$ and $-1/12$, and require an additional dimension 6 operator, $\sum_{\mu,\nu,\rho} \mathrm{Tr}(D_\mu F_{\nu\rho} D_\mu F_{\nu\rho})$, represented on the lattice by a 6-link generalized parallelogram along the edges of a simple lattice cube.[5]

We have used this one-loop Symanzik-improved pure gauge action (see ref.[5] for details) with tadpole-improved coefficients as described in[6] to generate our gauge configuration ensembles. Using this improved gauge action ensures that the discretization effects, which are the subject of our study, are dominated by the fermions even with the improved fermions.

Wilson fermions with the commonly used normalization are given by the action

$$\begin{aligned} S_W &= 2\kappa \sum_x \bar{\psi} \left[\not{D} - \frac{1}{2}\Delta + m \right] \psi \\ &= \sum_x \bar{\psi}(x)\psi(x) + \kappa \sum_{x,\mu} \bar{\psi}(x) \left[(\gamma_\mu - 1)\, U_\mu(x)\psi(x+\mu) \right. \\ &\quad \left. - (\gamma_\mu + 1)\, U_\mu^\dagger(x-\mu)\psi(x-\mu) \right] \end{aligned} \tag{1}$$

where $\kappa = 1/(8+2m)$. The dimension 5 (irrelevant) Wilson term $\bar{\psi}\Delta\psi$ was introduced to lift the doublers, present with naive fermions on the edges of the Brillouin zone. But this term leads to $\mathcal{O}(a)$ discretization errors for the Wilson fermions.

In the spirit of the Symanzik improvement program, the $\mathcal{O}(a)$ effects can be canceled by including another dimension 5 operator

$$- c_{SW} \frac{\kappa}{2} \psi(x) \sigma_{\mu\nu} i F_{\mu\nu}(x) \psi(x) \tag{2}$$

into the action Eq. (1) as noted by Sheikholeslami and Wohlert.[7] On the lattice $F_{\mu\nu}$ is represented by a clover leaf of plaquettes around site x. For this reason these improved fermions are often referred to as clover fermions. The clover coefficient c_{SW} is equal to 1 at tree level and equal to $1/u_0^3$, where $u_0^4 = \langle \mathrm{Tr} U_p/3 \rangle$, with tadpole improvement. It has been shown that, with Wilson pure gauge action, the one-loop coefficient is dominated by the tadpole contribution.[8] With Wilson pure gauge action the clover coefficient has recently also been determined nonperturbatively.[9] Both the one-loop computation and the nonperturbative determination do not apply to the tadpole-improved 1-loop Symanzik pure gauge action used by us. We therefore used the tadpole-improved tree-level clover coefficient, $c_{SW} = 1/u_0^3$.

3 Comparing Clover with Wilson fermion spectroscopy

To test whether clover fermions have indeed reduced discretization errors as compared to Wilson fermions we made an extensive quenched spectroscopy computation. We generated 6 ensembles of pure gauge configurations with the tadpole-improved 1-loop Symanzik pure gauge action with lattice spacings, as measured from the string tension with $\sqrt{\sigma} = 440$ MeV, of 0.16, 0.18, 0.21, 0.26, 0.34, and 0.42 fm. All lattices have size $16^3 \times 32$ and hence our smallest system has spatial size $L_s \simeq 2.56$ fm. We therefore expect finite volume effects

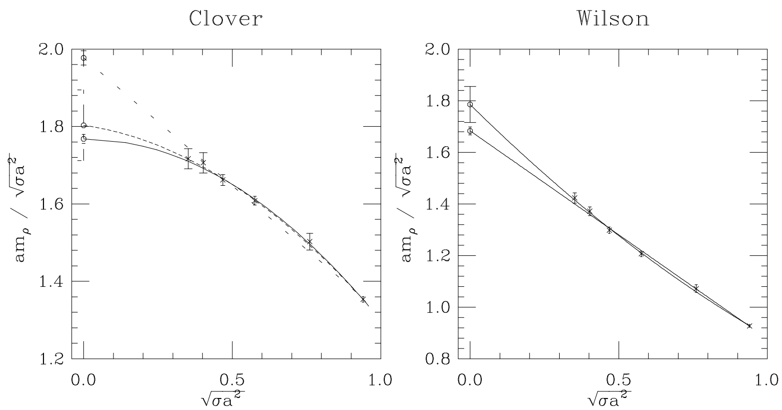

Figure 1: Scaling plot of $m_\rho/\sqrt{\sigma}$ versus the lattice spacing as measured in units of $\sqrt{\sigma}$.

to be very small. Except for this smallest system we used the method of '$Z(3)$' fermion sources[10] to increase the statistics at no extra computational cost.

For hadron measurements, we used correlated multi-state fits to multiple correlation functions as discussed in [11]. We computed quark propagators in Coulomb gauge at several κ values, with $m_\pi/m_\rho \gtrsim 0.5$, on each configuration ensemble. We used two gaussian source smearing functions with smeared and local sinks. The hadron masses were then extrapolated to the physical m_π/m_ρ ratio using correlated chiral fits

$$m_H = c_0 + c_2 m_\pi^2 + c_3 m_\pi^3 \qquad \text{for} \quad H = \rho, N, \Delta. \tag{3}$$

We actually used a cubic fit only for the clover data at the largest lattice spacing.

In Fig. 1 we show the ratio $m_\rho/\sqrt{\sigma}$ versus the lattice spacing for both clover and Wilson fermions. Also shown in the figure are scaling fits

$$R(a) = R(0) \ \left[1 + \mu_1 a + (\mu_2 a)^2\right] \tag{4}$$

to the dimensionless ratio. We call the scaling fit linear, when $\mu_2 = 0$, quadratic, when $\mu_1 = 0$, and mixed when both coefficients are allowed to vary. The confidence level for the different fits are listed in Table 1 and in Table 2 we give the scaling slopes for the rho, nucleon and delta for linear and quadratic scaling fits for Wilson and clover fermions, respectively.

For Wilson fermions both linear and mixed fits work well, but there is a significant $\mathcal{O}(a^2)$ contribution. For the mixed fit to $m_\rho/\sqrt{\sigma}$ we find $\mu_1 = 280$

Table 1: Confidence level, Q, of scaling fits, Eq. (4), to the ratio $R = m_\rho/\sqrt{\sigma}$.

Q	linear	quadratic	mixed
Wilson	0.45	$\sim 10^{-5}$	0.69
Clover	0.64	0.93	0.96

Table 2: Scaling slopes μ_1 for Wilson fermions (linear scaling fit) and μ_2 for clover fermion (quadratic scaling fit).

μ (MeV)	Wilson	Clover
m_ρ	215	225
m_N	115	160
m_Δ	170	200
J_{K^*}	70	190

and $\mu_2 = 160$ MeV with opposite signs. For clover fermions we can not exclude the linear scaling fit. However, the quadratic fit works very well, and in the mixed fit we obtain a linear coefficient compatible with zero: $\mu_1 = -49 \pm 68$ MeV. We also find that the quadratic fit for clover fermions is stable when dropping the points with largest lattice spacing, while this is not the case for the linear fit for Wilson fermions. We conclude that our clover results are compatible with $\mathcal{O}(a^2)$ discretization errors, while for Wilson fermions both $\mathcal{O}(a)$ *and* $\mathcal{O}(a^2)$ contributions appear significant. We finally note that the scaling slopes (see Table 2) are of reasonable magnitude, maybe even slightly smaller than the 300-500 MeV expected.

In Fig. 2 we compare our results on "improved glue" with a collection of recent quenched spectroscopy computations with clover,[12] Wilson [13]and staggered fermions.[14] A quadratic scaling fit to the staggered fermion data, and a mixed scaling fit to the Wilson data are also shown. It is satisfying to notice that the continuum extrapolations all agree within their statistical errors.

In Fig. 3, we plot the ratios m_N/m_ρ and m_Δ/m_ρ assuming $\mathcal{O}(a^2)$ scaling violations for clover fermions. For m_N/m_ρ we find a continuum value of 1.29(2) that is consistent with the GF11 result;[10] however, it is different than the experimental value of 1.22. For Wilson fermions a linear scaling fit gives a value inconsistent with the clover extrapolation and lower than the experimental value, while a mixed scaling fit gives a consistent continuum extrapolation, albeit with large errors. Both scaling fits give similar confidence levels, Q. For

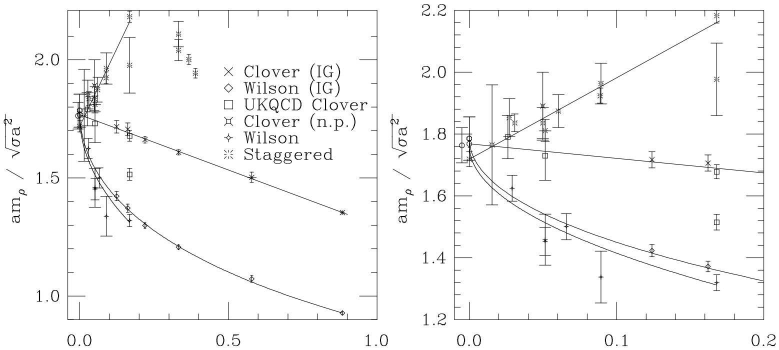

Figure 2: Scaling plot of $m_\rho/\sqrt{\sigma}$ versus a^2 for a collection of recent spectroscopy computations with clover, Wilson and staggered fermions. At right is a blowup of the small a region.

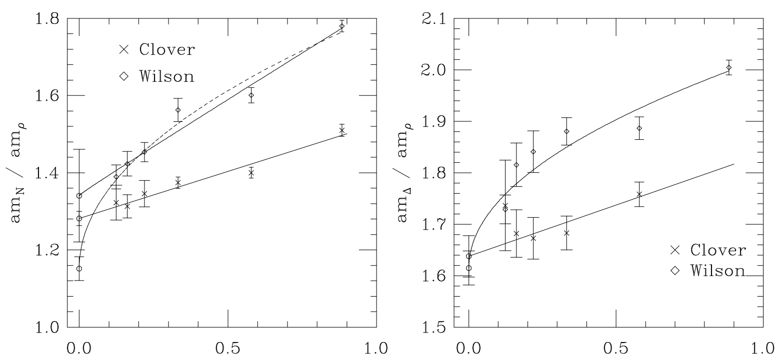

Figure 3: Scaling plots of m_N/m_ρ and m_Δ/m_ρ versus a^2 with quadratic scaling fit for clover fermions. For Wilson fermions are shown the linear (dashed) and mixed (full) scaling fit for the nucleon, and the linear scaling fit for the delta.

m_Δ/m_ρ we obtain a continuum value of 1.65(4) for clover fermions and, using only an $\mathcal{O}(a)$ ansatz, 1.61(3) for Wilson fermions. Both extrapolations agree, within errors, with the experimental value, 1.60.

4 Conclusions

We have made a comprehensive study of discretization effects for Wilson and $\mathcal{O}(a)$ improved clover fermions. This was done in the context of a quenched spectroscopy calculation on gauge field ensembles with lattice spacings varying from 0.42 to 0.16 fm. The gauge field configurations were created with an $\mathcal{O}(a^2)$ improved pure gauge action, ensuring that the leading discretization errors come from the fermions. We found the discretization errors for clover fermions to be compatible with being $\mathcal{O}(a^2)$, though numerically we could not exclude small $\mathcal{O}(a)$ contributions. For Wilson fermions we found that both $\mathcal{O}(a)$ and $\mathcal{O}(a^2)$ contributions are significant, making a reliable continuum extrapolation difficult. For the rho mass we found at our largest lattice spacing, $a = 0.42$ fm, scaling violations of $\sim 50\%$ for Wilson and $\sim 25\%$ for clover fermions. At our smallest lattice spacing, $a = 0.16$ fm, they decreased to $\sim 20\%$ and $\sim 3\%$, respectively. Therefore, for clover fermions we can make reliable continuum extrapolations with the extrapolation errors not exceeding a few percent.

We found that the scaling slopes are of reasonable magnitude, ~ 200 MeV. This suggests that an expansion in powers of the lattice spacing is reasonably behaved and converging, encouraging news for attempts of higher order improvements.[3]

Acknowledgments

This research was supported by DOE contracts DE-FG05-85ER250000 and DE-FG05-92ER40742. UMH would like to express his gratitude for the hospitality of the Zentrum für interdisziplinäre Forschung and the Fakultät für Physik of the University of Bielefeld during his stay in Bielefeld.

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